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Parimutuel Insurance for Hedging against Catastrophic Risk

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Abstract

This paper examines whether parimutuels can hedge risk-averse people against catastrophic losses. Two optimal stake choice models are constructed. In the first model, where stakes of other players are exogenous, the dynamic optimal hedge is obtained. In the second model, the equilibrium of stake is derived by maximizing the representative agent's expected utility. Given no transaction cost, participants in parimutuels are underinsured due to basis risk. The actual transaction cost for traditional insurance is found to be higher than the equivalent utility level implied by HuRLOs, suggesting that hedgers could be better off with HuRLOs than with traditional insurance.

Keywords: catastrophic risk, hedging, hurricanes, insurance, parimutuel

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1. Introduction

People and firms can insure against catastrophes such as hurricanes in the United Sates. However, for various reasons, this market does have limited capacity. This limitation is due to many factors; the high geographic concentration of insured risk; state regulatory strategies that constrain prices, and various types of transaction costs. To supplement the insurance market, a very different type of financial instrument has recently emerged for taking positions on hurricane landfalls; this is a parimutuel market place and the new instruments are known as HuRLOs (Hurricane Risk Landfall Options). While this structure resembles parimutuel horse race betting, the reasoning behind the opening of the HuRLO market was to provide further hedging capacity to those exposed to risk.

The new HuRLO instruments can be classified as an Insurance Linked Securities (ILS's), thus supplementing the existing instruments such as industry loss warranties, catastrophe options, and catastrophe bonds. These existing ILS's are targeted at insurance companies and large investors. Insurers use ILS's to provide hedge capacity (supplementary to reinsurance), and for portfolio diversification. And investors, such as hedge funds, are now treating natural catastrophe risk as a new asset class that has a potential for high returns and which has low correlation with existing asset classes. What is very different about the HuRLOs, is that while they can be used by similar stakeholders (insurance companies and large investors), they is now being made available to smaller players such as individual householders and small businesses. The reason for targeting these smaller players is to supplement existing insurance protection which may be limited. Such limitations can occur in various ways such as large deductibles, the effects of regulatory cross subsidies built into premiums, or the failure of business interruption insurance to make payments unless the policyholder's own property is damaged.

On its face, a parimutuel does not seem an ideal instrument for hedging since it has inbuilt basis risk. Payments do not depend on whether the individual suffers a financial loss, only on a hurricane landfall occurs in the location chosen. However, we will show that, for quite reasonable parameters, these parimutuel "bets" might sometimes offer higher expected utility to risk-averse householders or small business owners, than traditional insurance. The issue boils down to a trade-off between basis risk and transaction costs. We will show that, for individuals with rather representative levels of risk aversion, HuRLOs might offer higher expected utility than traditional insurance, when the insurance transaction costs exceed 20-25%.

The parimutuel mechanism was invented by Pierre Oller in 1865 as a betting system which guarantees a fixed profit for bookmakers. Nowadays, the parimutuel wagering system is adopted in at least 37 nations worldwide with trading volume up to $100 billion per year. Since 2002, various investment banks[1] have also employed the parimutuel mechanism in trading options on economic statistics, such as the US non-farm payroll report, Euro-zone harmonized inflation, and speed at which the Fannie Mae mortgage pool receives prepayment. Most recently, on October 7th, 2008, an innovative commodity option that applies the parimutuel was launched by Weather Risk Solutions, LLC (WRS); it is called Hurricane Risk Landfall Options (HuRLOs)[2].

[1] Deutsche Bank and Goldman Sachs introduced the first Parimutuel Derivative Call Auctions of options on economic data releases, including employment, industrial production, economic growth, inflection, etc.

[2] In 2008, HuRLOs trade via the WRS electronic trading platform on the Chicago Mercantile Exchange Alternative Marketplace, Inc.'s (CME AM) exempt board of trade (EXBOT) and are cleared by the CME. Their prices are determined first based on the historical hurricane risk database and then

The HuRLO market provides individuals the opportunity to hedge against, or speculate on, the risk that a specific region in the Gulf of Mexico and on the East Coast of the U.S. between the Mexican and Canadian borders will either be first hit by a hurricane or no hurricane will occur during a year. The HuRLOs market opened in early January 2009, continues into the hurricane season, and closes when a hurricane makes landfall or no hurricane makes landfall before the middle of December among the covered regions[3]. As a hurricane approaches so close to a specific region that the National Hurricane Center (NHC) issues a hurricane warning, trading will be suspended until the hurricane makes landfall or the imminent threat of the hurricane landfall abates. The risk pools are separated for different series of a hurricane's landfall. For example, participants in Series 1 (2) HuRLOs involve the risk of the landing of the first (second) hurricane.

In order to facilitating the transaction of HuRLOs, WRS also provides market-based and forecast-based landfall probabilities for each covered region on its website[4]. Market-based probabilities that reflect the price for a single option will change with each purchase in a particular risk pool. Forecast-based probabilities are estimated based on historical data and current weather conditions. When no current hurricane is identified, these estimates simply reflect historical information; otherwise, these probabilities are based on current NHC forecast data.

Both hedgers who possess assets in the hazard-prone areas, and speculators who would like to bet on which covered area will first be hit by a hurricane, may be interested in trading HuRLOs. The participation of speculators enhances the market liquidity and expands risk pools. While, in this paper, we only address hedgers, a paper by Meyer et al. (2008) examines the parimutuel market with only speculators. They test empirically the HuRLO market in a controlled laboratory experiment, where participants are allowed to buy HuRLOs to maximize their profits in a simulated hurricane season. With limited trading experience, market prices converge quickly to objective probabilities of hurricane landfalls, suggesting that these trades act efficiently. Most potential investment anomalies[5] are not empirically verified. However, the boomerang bias, where the "no landfall" option is overpriced immediately after a hurricane passes through a specific region, is observed according to the experimental results.

The paper proceeds as follows. Section 2 presents the introduction of the parimutuel mechanism. Section 3 compares parimutuels with traditional insurance. In Section 4, given that stakes from others are exogenous, we formulate a model for a risk-averse individual who would like to hedge against potential hurricane risk by

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interact dynamically by trading decisions made by all participants via a mathematical adaptive control algorithm that adjusts in a way that makes the selected outcome in the last trade more expensive and other outcomes less expensive. Detailed procedures of the algorithm are presented in Meyer et al. (2008).

[3] In 2008, HuRLOs market began in the latter part of the hurricane season.

[4] http://www.weatherrisksolutions.com/

[5] Investment anomalies examined include procrastination biases, distorted beliefs about probabilities, speculative bubbles, and false-alarm biases. Procrastination represents investors' preference for purchasing HuRLOs until a storm is formed and threatens a specific area. This bias could decrease the size of a mutualized risk pool and increase the uncertainty to returns. Although the HuRLO market provides objective landfall probability for each area, a distorted belief about probabilities could arise due to the perception bias of investors. Speculators believe that the higher price of HuRLOs in a specific area relative to its objective probability could reflect the private information held by other investors, further raising the option price and creating bubbles to the price. False-alarm bias describes the situation where no landfall in a specific area that was thought to be likely would discourage later investment in that area.

parimutuels and analyze his optimal stake choice. Section 5 considers the case in which a risk-averse representative hedger uses parimutuels to insure his potential loss from a hurricane. The equilibrium of parimutuel stake and the comparative static analysis are provided. Section 6 summarizes the pros and cons of parimutuel insurance. Section 7 discusses the role of speculators. Section 8 estimates the equivalent transaction costs of traditional insurance relative to HuRLOs. Section 9 concludes.

2. The Parimutuel Mechanism

The parimutuel mechanism is a betting system where bettors wager on one of the exhaustive and mutually exclusive possible outcomes before the race begins. As the outcome of the race is realized, the total stakes of bettors are distributed among the bettors who wager on the winner of the race, in proportion to the initial stakes. Bettors who wager on other horses lose their stakes. In practice, bookmakers usually deduct a certain percentage of the total amount of stakes as taxes and transaction fees, called the track take. Suppose there are S exhaustive and mutually exclusive outcomes with m_s wagers on outcome s, where $s=1, 2\ldots, S$. As outcome k turns out to be the winner, each bettor who wagers on outcome k receives $\sum_{s=1}^{S} m_s \Big/ m_k$ for each dollar he wagered; while bettors who wager on outcomes other than k receive nothing. To put it differently, the winning bet is refunded with his original stake and an extra return stemming from equally sharing the stakes of the losers. This extra return, $\sum_{s \neq k} m_s \Big/ m_k$, also known as odds, shows the net return that will be paid out to the bettor, should he win, relative to his stake.

A framework of the mathematical principles of parimutuel pricing and their implications is presented in Baron and Lange (2007). These include the arbitrage-free and the self-hedging principles. The arbitrage-free principle describes two conditions under which parimutuel participants can not profit without taking any risk: the price of each state is positive, and the prices of all states are summed to be one. The former condition precludes the possibility of making profit by wagering on any single state, whereas the latter rules out the possibility of making profit by wagering on all states simultaneously. The self-hedging principle states that all payouts of the parimutuel game come from the initial wagers. Without a transaction fee and tax, the parimutuel mechanism is a zero-sum game for participants. This feature appeals to market makers, who bear no risk. By these two principles, the price of the state s claim (or the implied probability of state s) is the proportion of stakes wagered on the state s relative to the stakes wagered across all states. Given that the total stakes across all states are the same, the relative stakes wagered on these states reflect the relative prices of the state claims. More stakes wagered on one state relative to another represent a higher price of the state relative to the other. This property is referred to as the relative-demand pricing since the stakes wagered on a state claim just reflect its relative demand.

Because horse racing was the earliest and the most popular wagering system to adopt the parimutuel mechanism, we demonstrate the general procedures on horse racing. The wagering period begins 20 to 30 minutes before the upcoming race starts and closes as soon as the race starts. During that period, bettors submit their target horse and wagers to bet. Payoff to the winning bet is announced after the race is finished and the official results are finalized. The odds, standing for the bettor's net returns on the specific horse, are inversely related to the state prices (or implied probability of that state). If all other conditions remain constant, the more wagers there are on one horse, the lower the odds are against that horse, but the higher the odds are against the other horses. Typically, the odds are exhibited on the track tote board in the form of odds to 1. During the wagering period, odds change over time with incoming wagers on different horses. Only the final odds, revealed at the end of the wagering period, reflect the net returns of the bets on horses. In this sense, bettors do not exactly know the claim price (or payoff) when they place their bet, and only find out when the wagering period closes.

In addition to wagering on a horse to win, bettors can wager on a horse to place or to show, where the bettors win the game if their target horse ends up in the top two or three positions, respectively. In practice, the win, place, and show wagers are pooled separately. This separate pooling creates arbitrage opportunities and reduces liquidity. As a result, extensive studies have been devoted to searching for systematic strategies to beat the parimutuel game[6].

Like capital markets, wagering markets contain a large number of participants and a variety of information sources. In addition, they have a well-defined termination point where eventual payoffs are determined. This feature offers economists an ideal environment to examine market efficiency and to observe human behavior under conditions of uncertainty. In contrast, security prices in capital markets change continuously and are affected by various uncertain factors, such as the future cash flow of firms and relative demand/supply structure in the markets.

Several market anomalies[7] have been observed in examining the market efficiency hypothesis of wagering markets. The most common one is the favorite-longshot bias, in Griffith (1949) and subsequent papers[8]. This bias is characterized by favorites (horses with short odds to win) that win more frequently than the odds predict and longshots (horses with long odds to win) that win less frequently than the odds predicts.

Consistent with the convex empirical utility function constructed by Weitzman (1965) and Ali (1977), the favorite-longshot bias reflects the risk-seeking behavior of participants in wagering markets. Since the expected returns on the favorites are higher than those on the longshots, bettors could make profits by wagering on favorites rather than on longshots. Although recent papers[9] do not empirically support the favorite-longshot bias in the US and other countries, a great number of theories have been formulated to explain this phenomenon[10]. Basically, these theories can be

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[6] With a high track take, approximately 15-25% of aggregate wagers, making a profit based on the odds and information inside the wagering pools seems to be a daunting task. However, Hausch, Ziemba, and Rubinstein (1981), Hausch and Ziemba (1985), and Ziemba and Hausch (1987) discovered profitable strategies, widely known as the "Dr. Z system", based on their finding that horses likely to win are underbet in place and show wagers. Asch, Malkiel, and Quandt (1984, 1986) and Asch and Quandt (1986) developed another strategies based on their finding that late bets contain information about the horses' eventual ranking. The more informed bettors tend to make wagers at the end of the wagering period since an early bet would adversely affects the parimutuel odds. In recent years, the frequent use of those strategies has reduced the effectiveness significantly.

[7] Three empirical regularities are summarized in Ottaviani and Sorensen (2005b): the puzzle of early betting, late informed betting, and favorite-longshot bias.

[8] Hausch, Lo, and Ziemba (1994) summarized the empirical articles that provide evidence for favorite-longshot bias.

[9] Busche and Wall (2000), Busche and Hall (1988), Busche (1994), Gandar, Zuber, and Johnson (2001), Bjorkman and Bukszar (2003) do not support favorite-longshot bias based on empirical evidence.

[10] Griffith (1949) ascribed to individuals' systematic undervaluation of favorites and overvaluation of longshots. Ali (1977) suggested that risk-seeking bettors are willing to accept lower returns for longshots. Hurley and McDonough (1995) proposed that a considerable track take reduces informed bettors' arbitrage opportunity, leading to few bets on favorites. Shin (1991;1992) argued that fixed-

categorized as two sets: risk-loving utility functions and misperceptions of probabilities. Based on the empirical tests of Snowberg and Wolfers (2010), the favorite-longshot bias is more likely driven by the misperceptions of probability, as suggested by the Prospect Theory, rather than by risk-loving utility.

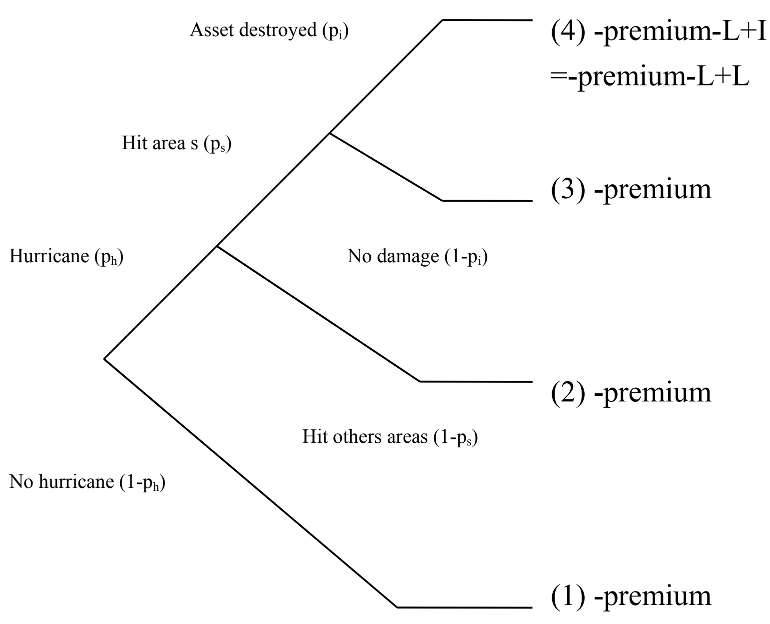
3. Comparing Parimutuels and Traditional Insurance

Parimutuels aggregate purchases into a mutualized risk pool and reallocate risks among all participants; as indeed does insurance. In order to compare the cash flows of parimutuel participants and those of traditional insurance policyholders, we divide the event space into four disjoint outcomes:

(1) No hurricane occurs
(2) A hurricane hits areas outside the target area
(3) A hurricane hits the target area but causes no damage to the policyholder's asset
(4) A hurricane hits the target area and destroys the policyholder's asset

Cash flows of traditional policyholders and cash flows of parimutuel participants in these disjoint outcomes are depicted in Figure 1 and Figure 2, respectively. Premium denotes the premium of the traditional insurance; L is the potential loss; x^* is the parimutuel stake; I represents the indemnity. Thus, net indemnity is simply $(I-x^*)$.

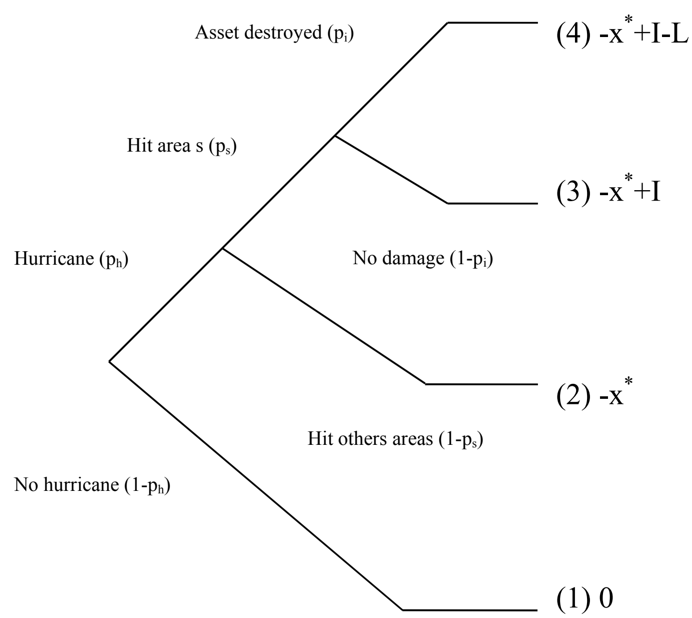
Figure 1: Cash Flows of Traditional Policyholders:



odds-betting bookmakers would prevent losses of the informed bettors. Ottaviani and Sorensen (2005b) noted that with a large number of bettors possessing some private information, posterior odds are more extreme than market odds, resulting in the bias. This paper also compared the merits and drawbacks among the theories described above.

For outcomes (1)-(4), the net cash flows of traditional insurance are the same if I=L. The indemnity is received by policyholders only when the actual loss is incurred (i.e., outcome (4)), but the indemnity offset the actual loss[11]. Thus, cash flows for policyholders of traditional insurance are negative (i.e., the insurance premiums) across four outcomes.

Figure 2: Cash Flows of Parimutuel Participants:



Cash flows of parimutuels are summarized as follows.

- In outcome (1), the cash flow is zero.

Stakes are returned to participants if no hurricane occurs.

- In outcome (2), the cash flow is a negative parimutuel stake.

Parimutuel stakes are paid out; no indemnity is obtained by the participants since the hurricane does not hit the target area.

- In outcome (3), the cash flow is the net indemnity, or the indemnity minus the parimutuel stake.

Parimutuel stakes are paid out; the indemnity is received since the hurricane hit the target area; no loss is incurred because the hurricane did not destroy the participant's asset.

- In scenario (4), the cash flow is net indemnity minus the loss.

Parimutuel stakes are paid out; the indemnity is received since the hurricane hit the target area; the loss comes from the hurricane's destruction of the participant's asset.

[11] Based on Mossin's theorem, if proportional insurance is available at a fair price, without loading factor and default risk, full insurance is optimal for a risk-averse individual.

A comparison of cash flows between parimutuel participants and insurance policyholders indicates that in outcomes (1) and (3), parimutuels are better than traditional insurances, but in outcomes (2) and (4), there is no monotone relation. For outcomes (1) and (3), this result is straightforward. The cash flows of parimutuel participants are zero in outcome (1) and positive in outcome (3) [12], respectively, whereas the cash flows of insurance policyholders are both negative. For outcome (2), the net cash flow depends on the relative size of the parimutuel stake and the insurance premium. If the insurance premium is greater than the parimutuel stake, parimutuels are better. For outcome (4), parimutuels are better whenever the insurance premium is greater than the loss minus the net parimutuel indemnity, i.e., $premium > L - \left(I - x^*\right)$. We use the following two examples to illustrate the comparison of cash flows in outcomes (2) and (4). In the first example, suppose that the probability of a hurricane's occurrence is very small and the insurance premium is less than the parimutuel stake[13]. The parimutuel is worse than the traditional insurance under outcomes (2) and (4). In the second example, suppose the probability of a hurricane's occurrence is high enough so that the insurance premium is greater than the parimutuel stake[14]; the parimutuel is better than the traditional insurance under outcome (2). Furthermore, if the probability of a hurricane's occurrence is higher such that the insurance premium is greater than the loss minus net indemnity, i.e., $premium > L - \left(I - x^*\right)$, the parimutuel is better in all outcomes.

4. Optimal Parimutuel Stakes

Suppose a risk-averse individual participates in the parimutuel mechanism to hedge against his potential loss from a hurricane in area *s* without transaction fees and taxes. We will refer to such hedging as "parimutuel insurance". *W* is the initial wealth of the individual, *L* is the loss that the individual will bear if a hurricane hits area *s* where his asset is located, p_h is the probability of the occurrence of a hurricane, p_s is the conditional probability of a hurricane hitting area *s* given that it actually occurs, p_i is the conditional probability the hurricane destroys the individual's asset in area *s* given that it hits area *s*. Here, all probabilities and conditional probabilities are assumed to be objective and are known to the individual. "*x*" is the stake that the individual places to hedge against the potential damage of his asset in area *s* under the parimutuel mechanism. $m_s(x)$ is the total stakes collected from those who purchase parimutuel insurance on area *s* while $M(x)$ represents the total stakes of individuals across all areas.

Parimutuels define the payout over an exhaustive set of outcomes. To satisfy this property, the event space is defined to include a set of counties where the first landfall may occur, plus a null event where no hurricane occurs in any of the counties. If the hurricane does not occur, the stakes gathered are distributed proportionally to each participant based on the initial stake[15].

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[12] Net indemnity is positive; otherwise, nobody has the incentive to purchase insurance.

[13] As shown in Section 4, parimutuel stakes are not affected by the probability of a hurricane's occurrence, but the insurance premium decreases with the lower probability.

[14] Similar to footnote 14, parimutuel stakes are not affected by the probability of a hurricane's occurrence, but the insurance premium increases with the higher probability

[15] This setting is different from that in the HuRLO market, where the exhaustive and disjoint outcomes include 78 regions with a potential hurricane threat and one "no landfall" outcome. Participants can also place stakes on the "no landfall" outcome. If the no hurricane makes landfall during a year, participants who place stakes on "no landfall" share all stakes based on their initial stakes.

We further define the odds and their relation with the payoff to parimutuel participants if the hurricane hits area *s*. The final odds against area *s* and the total stakes on areas outside area *s* over the total stakes on area *s* are denoted as $O_s(x)$

$$O_s(x) = \frac{M(x) - m_s(x)}{m_s(x)} = \frac{M(x)}{m_s(x)} - 1$$

The payoff of the stake is as follows:

$$payoff_s(x) = \begin{cases} \dfrac{M(x)}{m_s(x)} & if \;\; the\ hurricane\ hits\ area\ s \\ 0 & o.w. \end{cases}$$

The ratio of the total stakes in all areas to the stakes in area *s* is simply the payoff if the hurricane hits area *s*. Thereby, the relation between the odds against area *s* and the payoff when area *s* is hit can be exhibited as $O_s(x) = \frac{M(x)}{m_s(x)} - 1 = \frac{1}{P_s(x)} - 1$, where $P_s(x)$ is the price of parimutuel insurance in area *s*, which is a function of *x* since the price alters as the individual places his stake. Furthermore, given the individual's stake on area *s*, *x*, the net indemnity of the parimutuel stake if the hurricane hits area *s*, is denoted as $NI_s(x)$. The relations among the net indemnity, the odds, the payoff, and the price of parimutuel claim are thus shown as $NI_s(x) = O_s(x) \cdot x = \left(\frac{M(x)}{m_s(x)} - 1 \right) \cdot x = \left(\frac{1}{P_s(x)} - 1 \right) \cdot x$

Our goal is to obtain the optimal parimutuel stake that the individual chooses during the wagering period. In this section, when an individual places this stake, the stakes of other participants across all areas are assumed to have been revealed to the public so that the individual has perfect information on odds across all areas prior to his decision. This assumption is equivalent to a situation where the individual places his stake at the end of the wagering period after all other participants have placed their stakes. Although this assumption seems to be unrealistic, we start with a simple case. The functional forms of $m_s(x)$ and $M(x)$ are further specified as $m_s(x)=m_0+x$, $M(x)=M_0+m_0+x$, where m_0 denotes the total stakes on area *s* placed by other participants, and M_0 denotes the total stakes outside area *s*. In order to derive the optimal stake, the individual maximizes his own expected utility function:

$$\underset{x}{Max} \;\; E[U(\widetilde{W})]$$

$$= (1 - p_h) \cdot U(W - x + \frac{M(x)}{M(x)} \cdot x) + p_h \cdot (1 - p_s) \cdot U(W - x)$$

$$+ p_h \cdot p_s \cdot (1 - p_i) \cdot U(W - x + \frac{M(x)}{m_s(x)} \cdot x) + p_h \cdot p_s \cdot p_i \cdot U(W - L - x + \frac{M(x)}{m_s(x)} \cdot x)$$

$$= (1 - p_h) \cdot U(W) + p_h \cdot (1 - p_s) \cdot U(W - x)$$

$$+ p_h \cdot p_s \cdot (1 - p_i) \cdot U(W + O_s(x) \cdot x) + p_h \cdot p_s \cdot p_i \cdot U(W - L + O_s(x) \cdot x)$$

Yielding a F.O.C.

$$\frac{1 - p_s}{p_s} \cdot \frac{U'(W - x)}{(1 - p_i) \cdot U'(W + O_s(x) \cdot x) + p_i \cdot U'(W + O_s(x) \cdot x - L)} = \frac{\partial \left(NI_s(x) \right)}{\partial x}$$

It is straightforward to confirm that the second order condition is negative[16], and that x^* satisfying F.O.C. is a global maxima, and hence, the optimal parimutuel insurance stake for the individual. The F.O.C. can also be presented as: $\frac{\partial x}{\partial NI_s(x)} = \frac{p_s}{1-p_s}\frac{E_i[U'(W_s)]}{U'(W_{ns})}$, where W_{ns} is the individual's wealth when a hurricane hits an area outside area s, and W_s is the individual's wealth when a hurricane hits area s no matter whether it causes damage to the individual's asset in area s or not. If the individual's stake does not influence the odds against area s[17], the left hand side of the F.O.C. represents the reciprocal of odds against area s (or the ratio of the conditional probability of a hurricane hitting area s to the conditional probability of a hurricane not hitting area s), i.e. $\frac{\partial x}{\partial NI_s(x)} = \frac{1}{O_s(x)}(or\ \frac{p_s}{1-p_s})\ if\ \frac{\partial O_s(x)}{\partial x} \approx 0$. Replacing the left hand side of F.O.C. with the relative conditional probabilities, the F.O.C. becomes: $E_i[U'(W_s)] = U'(W_{ns})$.

The result shows that the optimal stake will equate the marginal cost of a net payoff in state "s", $\frac{\partial x}{\partial NI_s(x)}$, with the ratio of the expected marginal utilities in the "payoff" state, "s" and the "no payoff" state, "ns". The net indemnity in state s depends on the odds against area s. If the impact of the individual's stake on area s is trivial, or equivalently, if the odds against area s are hardly influenced by the individual's stake, the relative marginal utility is simply the reciprocal of the odds against area s, or the ratio of the conditional probability of a hurricane hitting area s and the conditional probability of a hurricane not hitting area s. In this case, the optimal stake can be solved by equating the expected marginal utility in the payoff state, "s", with the marginal utility in the "no payoff" state, "ns".

Basis risk is reflected in the expected marginal utility when a hurricane hits area s, i.e., $E_i[U'(W_s)]$, of the F.O.C. If the conditional probability of a hurricane destroys the individual's asset in area s, given that it hits area s, is further set to be one, or equivalently, when a hurricane hits the target area, all assets in the area will be destroyed, the optimal stake choice for the individual in the parimutuel is the same as the optimal amount of insurance in traditional insurance. Thus, parimutuel insurance can be differentiated from traditional insurance by the basis risk that the individual will encounter.

The optimal parimutuel stake derived by the F.O.C. for the individual is determined by six factors, i.e., $x^*(W, m_0, M_0, p_s, p_i, L)$:

- The initial wealth of the individual, the stakes on area s by other bettors
- The stakes outside area s
- The conditional probability of area s being hit by a hurricane if the hurricane should occur
- The conditional probability of the individual's asset being destroyed if a hurricane hits area s
- The potential loss

[16] The signs of the first and the second derivatives of the net indemnity if the hurricane hits area s with respect to the parimutuel stake are proven in Appendix 1.

[17] Generally, this condition is true if the market is comparatively well developed or the mutualized risk pool is relative large.

It is of interest that the probability of a hurricane's occurrence, p_h, does not have any impact on the optimal parimutuel stake. The rationale is intuitive: the redistribution of parimutuel stakes among participants occurs only when a hurricane hits one of the *S* areas. If no hurricane occurs, these stakes are given back to the participants without any transaction fee or tax. Moreover, the final odds against area *s* play a key role in determining the optimal stake. In parimutuels, odds, which are crucial to determine the payoff, fluctuate through time until the close of the wagering period. As an additional stake is placed on one area, the odds against all areas change accordingly. The individual thus has an incentive to postpone his stakes until more information about the odds are revealed to decide his optimal stake.[18]

The above optimal decision is treated as a static problem. However, the parimutuel market evolves over the year with the odds changing. Thus, this should be a dynamic hedging problem. The static optimal hedge will be achieved when trading closes based on available information. Furthermore, how to use our results to implement dynamic hedging in the HuRLOs market is shown as follows. If the impact of an individual stake on odds is negligible, the optimal stake can be determined by the following equation:

$$\frac{1-p_s}{p_s}\cdot\frac{U'(W-x^*)}{(1-p_i)\cdot U'(W+O_s(x)\cdot x^*)+p_i\cdot U'(W+O_s(x)\cdot x^*-L)}=O_s(x)$$

which is modified from the F.O.C. As can be seen, the optimal parimutuel stake is a function of several parameters, including the odds and the conditional probability of area *s* being hit. In trading the HuRLOs, the investors know the odds, $O_s(x)$[19] (from market-based probabilities) and the conditional probabilities, p_s[20] (from forecast-based probabilities) at any point in time. Both market-based probabilities and forecast-based probabilities in all covered regions are public information for investors and are available on the WRS website. Thus, given the optimal stake rule, hedgers can actually determine their hedging position with available information.

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[18] The timing of bettors' wagers, as indicated in Ottaviani and Sorensen (2005b), is determined by two opposing forces: the bettor's market power and the bettor's concern about information revelation. In a simultaneous-move game, bettors holding sizeable wagers and common information would bet early to prevent from adverse impact on odds while many small bettors with private information would place a late bet to conceal their private information and to grasp more information on the odds from other bets. Under the parimutuel mechanism, informed bettors prefer to postpone their wager until the last minutes because all bets are executed at the same final price.

[19] $O_s(x)=\frac{1}{\tilde{p}_s}-1$, where $\tilde{p}_s=\frac{\hat{p}_s}{1-\hat{p}_{nh}}$, $\hat{p}_s$ is the market-based probability of the first hurricane making landfall in region *s*, $\hat{p}_{nh}$ is the market-based probability of no hurricane making landfall in any of the covered regions. The market-based probabilities for all covered regions are available on the WRS trading website.

[20] $p_s=\frac{\bar{p}_s}{1-\bar{p}_{nh}}$, where $\bar{p}_s$ is the forecast-based probability of the first hurricane making landfall in region *s*, $\bar{p}_{nh}$ is the forecast-based probability of no hurricane making landfall in any of the covered regions. The forecast-based probabilities for all covered regions are available on the WRS trading website.

5. Equilibrium of Parimutuel Stakes

In this section, we analyze a risk-averse representative agent's optimal stake decision under the parimutuel mechanism. In this special case, S areas are at risk to be hit by a hurricane. Suppose all residents participate in the parimutuel insurance and own assets with the same market value in one of the S areas. There are n residents in each area. The hurricane hits each area with an equal probability. An individual in area s is taken to be a representative agent who has W dollars for his initial wealth, places the stake worthy of x in parimutuels, and losses L dollars if a hurricane hits his asset in area s. Since p_s are the same across all areas, $p_s = 1/S$. When a hurricane hits area s, the payout to the representative agent is the ratio of the total stakes to the number of residents in the area, i.e., $(S \cdot n \cdot x)/n = S \cdot x$. If no hurricane occurs, the total stakes are returned proportionally to the initial stake.

The objective is to maximize the expected utility of the representative agent:

$$\underset{x}{Max}\ E[U(\widetilde{W})]$$

$$= (1-p_h)U(W) + p_h \cdot (1-p_s)U(W-x) + p_h \cdot p_s \cdot (1-p_i) \cdot U(W+(S-1)\cdot x)$$

$$+ p_h \cdot p_s \cdot p_i \cdot U(W+(S-1)\cdot x - L))$$

Differentiating the objective function with respect to x yields the F.O.C.:

$$\frac{\partial E(\widetilde{W})}{\partial x}$$

$$= -p_h \cdot \frac{S-1}{S} \cdot U'(W-x) + p_h \cdot \frac{S-1}{S} \cdot (1-p_i) \cdot U'(W+(S-1)\cdot x)$$

$$+ p_h \cdot \frac{S-1}{S} \cdot p_i \cdot U'(W-L+(S-1)\cdot x) = 0$$

$$\Rightarrow U'(W-x^*) = (1-p_i)\cdot U'(W - x^* + \underbrace{S \cdot x^*}_{\geq 0}) + p_i \cdot U'(W - x^* + S \cdot x^* - L)^{21}$$

For the F.O.C. to hold with $S \cdot x^* \geq 0 \,\&\, U'' < 0$, then $S \cdot x^* - L \leq 0$, (or equivalently, $x^* \leq L/S$), must be satisfied. Thus a parimutuel will, in general, provide a payout to the representative agent that is less than the potential loss. This result rests on the presence of basis risk. The parimutuel pays on the occurrence of loss in area s. However, conditional on area s being hit, the probability that the stakeholder suffers a loss, p_i, is less than one. Thus, the parimutuel provides a windfall gain to an individual even if he/she suffers no loss. In effect, the parimutuel introduces background risk. We know, in general, that the presence of background risk will upset the normal optimality condition for full (actuarially fair) insurance. Only in the special case where p_i=1; (equivalently n=1 or only one stakeholder in area s,) will the F.O.C. show that a full indemnity, $x^* = L/S$ is optimal, consistent with Mossin's theorem[22]. Thus, parimutuel insurance intrinsically leads to underinsurance even in the case of no transaction fee or tax.

However, for the general case where $p_i < 1$, the parimutuel offers a payout on the hurricane hitting the target area s whether or not the individual actually has a loss. In contrast, the mutual insurance rules out the possibility of the windfall gain, and hence the policyholder's decision only depends on balancing marginal utilities in two states: when the hurricane hits area s and when the hurricane hits other areas, i.e.,

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[21] The negative sign of the second derivative is easily verified.

[22] Mossin's Theorem (1968) is often considered the cornerstone result of insurance economics.

$U'(W-x^*)=U'(W-L+(S-1)\cdot x^*)$. A risk-averse mutual insurance policyholder will choose full insurance, i.e., $x^*=L/S$, since the insurance premium genuinely reflects the expected insured losses.

We further explore the comparative statics on the equilibrium of the parimutuel stake regarding the underlying four parameters; $x^*(W, p_s, p_i, L)$[23]. The summary of our analysis is as follows. The sensitivities of the equilibrium of the parimutuel stake with respect to the loss caused by a hurricane, with respect to the conditional probability of a hurricane hitting the target area given that it occurs, and with respect to the conditional probability of an individual's asset being destroyed given that the hurricane hits the target area, are all positive, i.e., $\partial x^*/\partial L>0, \partial x^*/\partial p_s>0, and\ \ \partial x^*/\partial p_i>0$.

The relation between the parimutuel stake and the conditional probability of a hurricane hitting area s suggests that the sensitivity of the equilibrium of the parimutuel stake with respect to the number of hurricane-prone areas is negative, i.e., $\partial x^*/\partial S<0$. The basic logic is simply balancing between the expected marginal utilities for two states: when a hurricane hits the target area and when a hurricane hits areas outside the target area. First, ceteris paribus, an increase in the number of hurricane-prone areas decreases the expected marginal utility when a hurricane hits the target area by increasing the payoff to the target area. The equilibrium parimutuel stake is then reduced, not only to decrease the payout when a hurricane hits areas outside the target area, but also to reduce the net indemnity when a hurricane hits the target area. Through this adjustment, the expected marginal utilities in these two states are equalized.

Similar to the previous analysis but with a reverse direction, an increase in the potential loss (or the conditional probability of the asset being destroyed by a hurricane given that it hits the insured area), ceteris paribus, raises the expected marginal utility when a hurricane hits the target area. In order to balance off the relative expected marginal utilities, the equilibrium of the parimutuel stake moves upward both to enhance the net indemnity when a hurricane hits the target area and to reduce the stake payout when a hurricane hits the areas outside the target area.

Nevertheless, the sensitivity of the equilibrium parimutuel stake with respect to the initial wealth is ambiguous. We derive the condition under which the sign of the sensitivity is positive:

$$\frac{\partial x^*}{\partial W}>0 \Leftrightarrow U''(W-x^*)<(1-p_i)\cdot U''(W+(S-1)\cdot x^*)+p_i\cdot U''(W+(S-1)\cdot x^*-L).$$

The rationale for this result is also similar. As the initial wealth increases, expected marginal utilities in both states decline accordingly. If the above condition holds, the marginal utility when a hurricane hits areas outside the target area is reduced for a greater amount than the expected marginal utility when a hurricane hits the target area. To balance between expected marginal utilities in these two states, the equilibrium of the parimutuel stake has to adjust upward. Appendix 2 provides the detailed derivations of the comparative statics and the determination of their signs for reference.

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[23] For the same reason as was stated in the previous section, the likelihood of the occurrence of a hurricane does not affect the equilibrium of stake. The redistribution of overall stakes is only triggered by a hurricane's hitting one of the S areas.

6. Pros and Cons of Parimutuel Insurance

Parimutuels have several merits that can potentially ameliorate or even overcome the recent obstacles that are encountered by insurers. Here is our summary:

- The insurer takes no risk.
  This is the most critical advantage of parimutuels. The insurer acts as a bookmaker and redistributes money among policyholders. Parimutuels add no financial capacity constraints to insurers.
- Liquidity is enhanced.
  In trading catastrophe-linked derivatives[24], traders have to find a counterparty who is willing to take the underlying risk on the other side. If trading parties do not match in a market, high bid-ask spreads or thin trading volume will appear. Catastrophe futures[25] are such an example. However, in parimutuels, policyholders can bet on any adverse outcome in the risk pool without insurers' matching offers.
- There is less default risk.
  In parimutuels, all indemnity payouts come from the stakes of the participants. Since all stakes are collected before adverse events and insurers simply redistribute the stakes, policyholders and insurers have less incentive to default on the transaction.
- Windfall gain is possible in parimutuels.
  Parimutuels pay out when a hurricane hits the target area whether or not the policyholder actually has a loss. When the hurricane hits the target area without destroying the participant's asset, the participant shares the total stakes with other target bettors, even if he does not bear any loss.
- Costs are reduced for insurers
  Traditional insurers must have expertise to estimate the probability of a hurricane's occurrence, the probability of a hurricane hitting each area, and the potential losses for policyholders. Large insurers usually hire hurricane modeling firms[26] or build a team of experts from different fields[27] to approximate and hedge the risk they take. However, parimutuel insurers bear no risk, thus they are not required to manage the catastrophic risk.
- The contracts are more flexible.
  Traditional insurance generally specifies a fixed period, like annual or semiannual, as the insured period so that the premiums collected are enough to cover the potential indemnity. Parimutuels have more flexibility in setting insured periods. For instance, we can specify the insured period to be hurricane-prone seasons based on historical data. Policyholders do not have to pay premiums for the whole year.

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[24] CAT-linked securities, such as Industry Loss Warranty (ILW), CAT options, and CAT bonds first appeared in the capital markets in the 1980s and 1990s. In addition to traditional reinsurance contracts, these securities together with Sidecars[24], provide risk-transferring alternatives in ameliorating CAT shocks within the economy.

[25] Catastrophe Futures were introduced by the Chicago Board of Trade (CBOT) in 1992 after Hurricane Andrew.

[26] The scientific risk analysis and quantitative risk estimates of the catastrophe damage is developed by catastrophe-modeling firms. Those firms build catastrophe models, which use meteorology, engineering, and insurance underwriting data to estimate damage in different areas. Input information is based on historical tropical storms, building construction, and the impact on various structures under different wind speeds. Today, the three leading proprietary catastrophe modeling firms are Risk Management Solutions (RMS), AIR Worldwide, and EQECAT.

[27] Include meteorology, engineering, and insurance underwriting.

- Parimutuels ameliorate information asymmetry.
  In order to deal with adverse selection, insurers have to design mechanisms[28] to distinguish clients with different risk types. In parimutuels, however, payouts depend only on the area hit by the event not upon participant characteristics. Similarly, payouts are independent of actions of participants (other than choice of stake). Thus moral hazard is avoided.

On the other hand, parimutuel insurance has several drawbacks.

- Parimutuel participants are underinsured.
  In response to basis risk, parimutuel participants would naturally choose the optimal parimutuel stake that provides less than full coverage should the participant suffer a loss.[29] In contrast, policyholders of traditional insurance would choose full insurance.
- Participants do not know the payoffs (or prices) when placing stakes.
  The potential payoffs are determined by the final distribution of the total stakes, which will not be revealed until the end of the wagering period. This feature can be abated by postponing the wager as late as possible before the wagering period closes, but can not be eliminated.
- Late Information
  Transactions are not allowed during the period between the end of the wagering period and the eventual outcome is realized. When adverse events against an individual's bets occur after the wagering period, it is impossible to withdraw his stakes. If the underlying risk is highly sensitive to updated information over time, participants could not react to incoming information immediately, thus having a lock-in risk. However, it is unlikely that insurer would underwrite at this time. Participants also have incentives to postpone the timing of purchasing parimutuel insurances in order to wait for the most updated information to be revealed[30]. There is an offsetting effect, if there is secondary trade in parimutuel stakes after the wagering period closes. Since secondary trades will reflect new risk, then hedging strategies are possible at this late time.
- Basis Risk
  When the probability of hurricane occurrence is low, the parimutuel stake could be higher than the traditional insurance premium. In addition, cash flows of parimutuels are less than traditional insurance when the hurricane hits areas other than the target area and when the hurricane destroys the assets of the participants.
- Parimutuels can only be applied to mutually exclusive events.
  Parimutuel pricing principles restrict the outcomes to be collectively exhaustive and mutually exclusive. Collectively exhaustive property can be relaxed by including a no hurricane state, but a mutually exclusive condition

[28] The self-selection mechanism, in which insurers offer a menu of insurance contracts with various prices and quantities so that insurers can distinguish the risk types of policyholders by observing their choices. Related literature include Rothschild and Stiglitz (1976), Spence (1978), Dionne (1983), Kunreuther and Pauly (1985), Dionne and Doherty (1994), etc.

[29] The indemnity is insufficient to pay for the actual loss.

[30] The late informed betting phenomenon, as described in Ottaviani and Sorensen (2005b): late bets contain a great deal of information about the horses' eventual ranking in the parimutuel game. Also documented in Asch, Malkiel, and Quandt (1982), Gandar, Zuber, and Johnson (2001), and Ottaviani and Sorensen (2010).

can not be violated. In applying parimutuels, it is important to define the insured events to be mutually exclusive. In a HuRLOs scheme, although a hurricane could hit more than one of the covered areas, the trigger event is defined as the covered area first hit by a hurricane during a year.

7. The Role of Speculators

A useful of parimutuels is that one purchase on one outcome will improve liquidity for all other outcomes. Thus, adding speculators to our model may enhance liquidity. Speculation could play a somewhat neutral role in our analysis if the speculators placed their bets across counties in direct proportion to the stakes placed by hedgers. But it is difficult to see what advantage speculators would derive from such a passive strategy. More likely, they would take advantage of perceived mispricing by placing bets on outcomes where the objective probabilities differ from market-based probabilities. This would, reduce (increase) the payoffs to hedgers in counties where speculators take net long (short) positions.

In our previous analysis, we assume that hedgers use the objective probability to estimate the likelihood that a hurricane makes landfall in a county. Laypersons have difficulty in predicting the probability that a hurricane makes landfall and have to resort to the objective probability that provided by meteorologists. The aggregate subjective probability of the first landfall should thus approach the corresponding objective probability, supporting our usage of the objective probability instead of subjective probability in optimizing the utility. Moreover, the market-based probability simply reflects the relative demand in that county. Relative higher demand in the target county compared to the objective probability will raise the market-based probability of the target relative to other counties. In this case, the market-based probability of the target is higher than the objective probability of the target, whereas the market-based probability of other counties is lower than the objective probability of other counties. Speculators would thus arbitrage on the discrepancy between the objective probability and the market-based probability by placing more stakes in other counties except for the target. This behavior, through parimutuels, will decrease the market-based probability of the target and increase the market-based probability of other areas until reaching the equilibrium, where the market-based probability equals the objective probability for all counties. Thus, in the presence of speculators, our results in section 4 will be approaching the equilibrium.

If the market is in equilibrium, an additional stake on a specific outcome placed by a speculator will drive down the payoff of that outcome while push up the payoff of all other outcomes. Nevertheless, the expected payoff on each outcome is still kept at the same level as prior to the additional stake thanks to the parimutuel mechanism. Speculators can not make any profit by placing any bet that deviates from the equilibrium. Since speculators would not participate in the equilibrium, the assumption on the absence of speculators in our models is then justified.

8. Equivalent Transaction Costs of Traditional Insurance Relative to HuRLOs

If there were no transaction costs, the optimal hedge with a HuRLO would not provide full insurance because of the inherent basis risk. In contrast, with traditional (indemnity) insurance with no transaction cost, it is optimal to fully insure. However, with transaction costs, the choice between hedging with a HuRLO or with insurance, essentially boils down to a trade off between transaction costs and basis risk. Although we do not have estimates of the transaction costs of HuRLOs, we believe these to be very low relative to traditional insurance, and probably in the order of a 1-

2%. In this section we would like to provide rough estimates of the tradeoff between transaction costs and basis risk. We will do so by estimating the optimal HuRLO stake for an assumed level of transaction cost. We will then estimate the transaction cost for a traditional insurance contract that equates the expected utility with optimal insurance to the expected utility yielded by the optimal HuRLO stake. If, this actual transaction costs for traditional insurance are higher (lower) than the utility equivalent level, it follows that the individual will be better (worse) off with the HuRLO than with traditional insurance.

Suppose that an individual possesses total assets with $1,000,000 in value, including a house that is worth $500,000. If hurricane strikes the area where the house located and destroys the house, he will suffer a loss of $500,000. The utility of the individual can be represented by a simple power utility function with a constant relative risk aversion coefficient, a, i.e., $U(W)=\frac{1}{1-a}W^{1-a}$, where a is assumed to be 0.71. This risk aversion estimate should be reasonable based on the reviews of empirical findings in Chetty (2006) and Cardenas and Carpenter (2007). The former summarizes 33 sets of estimates of labor supply elasticities and calculates the implied coefficients of relative risk aversion ranging from 0.15 to 1.78 with mean 0.71 while the latter review coefficients from risk experiments in various countries and finds estimates between 0.05 and 2.57. Palm Beach and Monroe, both located in Florida, are the two target areas that we will focus on to illustrate the equivalent transaction cost relative to HuRLOs. For each area, there exist two sets of probability: one is forecast-based probability and the other is market-based probability. From actual trading data on a specific date in 2009, the two sets of probabilities are (p_h=0.0395, p_s=0.0228, p_i=0.1) and ($p^m{}_h$=0.2454, $p^m{}_s$=0.0595, p_i=0.1) in Palm Beach and (p_h=0.0395, p_s=0.0734, p_i=0.1) and ($p^m{}_h$=0.2454, $p^m{}_s$=0.0717, p_i=0.1) in Monroe. If a hurricane hits an area, we suppose that there is a one-in-ten chance that houses within the area will be destroyed (p_i=0.1). Thus, the optimal stake and the maximal expected utility in the HuRLOs market can be obtained by first collecting the probability of hurricane occurring (p_h) and the conditional probability of hurricane striking a specific area (p_s) from the HuRLOs market, inputting them into the expected utility of the individual, and then exercising optimization. The expected utility is similar to that is shown in section 4 with the odds derived by the conditional probability of hurricane striking the area, O_s=(1/p_s)-1.

The maximal expected utilities empirically obtained from HuRLOs are used to derive the equivalent transaction cost of the traditional insurance contract by assuming the individual is fully-insured and partially-insured, respectively. If the individual is fully-insured, the equivalent transaction cost simply reflects the basis risk of the specific HuRLO; however, if the individual is allowed to be partially-insured, the equivalent transaction cost balances the synthetic effects of the basis risk of the HuRLO with the individual's incentive to buy less than full insurance when insurance is costly.

Figure 3 shows the comparisons of the expected utilities for an individual between HuRLOs and traditional insurance in four cases with two dimensions, the area (Palm Beach/Monroe) and the type of probability (forecast-based/market-based), assuming no transaction cost in HuRLOs. For the same case, the left hand side exhibits the HuRLOs and the right hand side demonstrates the traditional insurance. Take the HuRLO in Palm Beach using forecast-based probabilities as an example. The optimal stake is $1,785.1 and the maximal expected utility is 189.4939. In order to achieve this expected utility in traditional insurance contracts given the constraint that the individual is fully-insured, the equivalent transaction cost is 20.91%. Thus if the actual transaction cost for insurance is greater than 20.91%, the individual will be better off accepting the basis risk of the HuRLO than full indemnity insurance. But, of course, full insurance is sub optimal with transactions costs. When being partial-insured is allowed (the optimal amount of insurance and the equivalent transaction cost are determined simultaneously), the optimal amount of insurance is 33.55% and the equivalent transaction cost is 33.2%. This equivalent transaction cost is higher than that of the full-insured case because the individual would choose to purchase less insurance in the presence of transaction costs.

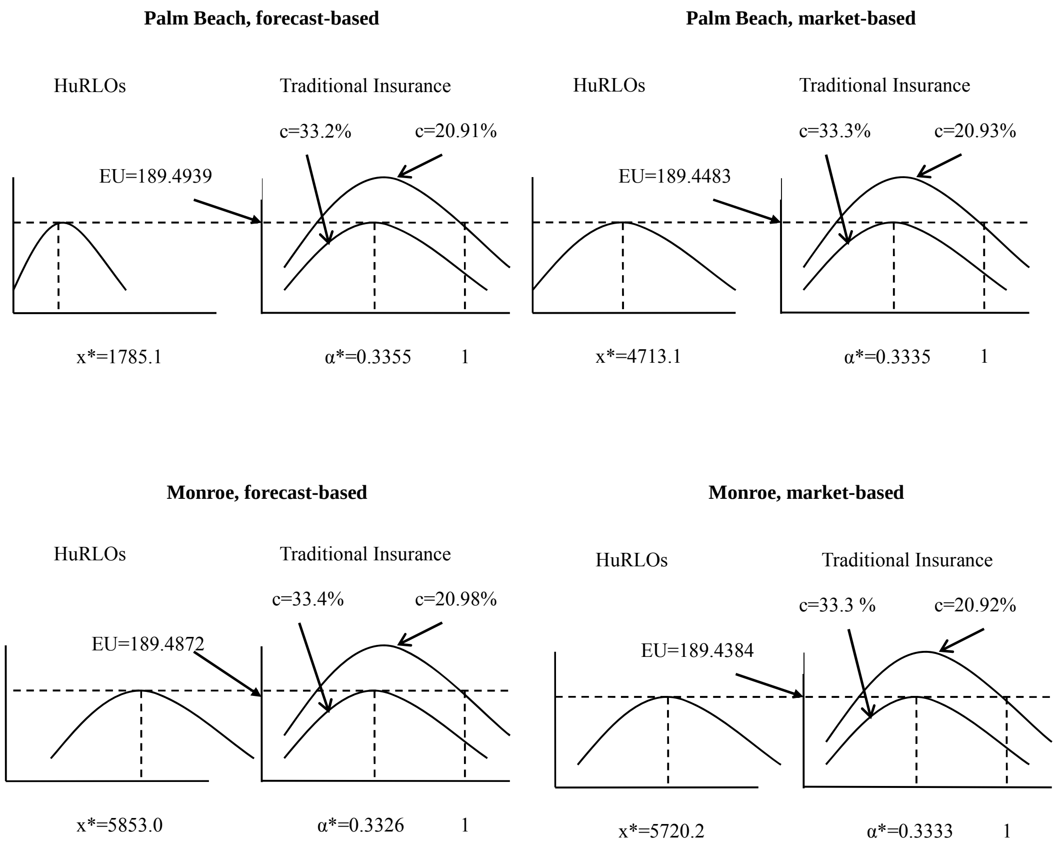

Figure 3: The Comparisons of the Expected Utilities for a Hedger between HuRLOs and Traditional Insurance in Four Cases. These diagrams show the comparisons of the expected utilities for a hedger between HuRLOs and traditional insurance in four cases: Palm Beach using forecast-based probability, Palm Beach using market-based probability, Monroe using forecast-based probability, and Monroe using market-based probability. For the same case, the left hand side exhibits the HuRLOs, and the right hand side demonstrates the corresponding traditional insurance. The vertical axes are the expected utility. The horizontal axis is the amount stake the individual would place (x) in HuRLOs while it is the proportion of insurance relative to full insurance (α) in traditional insurance. On the right hand side, α=1 represents the individual being fully-insured and c denotes the equivalent transaction cost in traditional insurance contract relative to the corresponding HuRLOs. The expected utility of the individual in traditional insurance will move downward with higher transaction costs. The upper curve constrains the individual to be fully-insured while the lower curve allows the individual to be partially-insured. Transaction costs in HuRLOs are assumed to be **0%** for each stake the individual placed.

According to the statistics of premiums and losses of 2009 top 25 companies in property and casualty insurance industry, collected by NAIC (National Association of Insurance Commissioners), the ratio of losses to premiums is approximately 60% (See Table 1, this ratio is between 59.27% to 60.94% by different definitions of losses). Empirical transaction costs of traditional homeowners multiple peril insurance are approximately 67% ((1/60%)-1) of the expected loss, which include administrative, marketing and claims processing costs, taxes, and cost of capital. The equivalent transaction costs relative to HuRLOs in the four cases range from 33.2% to 43.2%, depending on transaction costs in HuRLOs (0%-2%). Thus, the actual transaction costs for traditional insurance (67%) are higher than the utility equivalent level (33.2%-43.2%). This result suggests that HuRLOs, in spite of the inherent basis risk, provide a more effective hedging mechanism than traditional insurance for homeowners because the transaction cost of HuRLOs is very low (less than 2%) relative to traditional insurance.

PROPERTY AND CASUALTY INSURANCE INDUSTRY
2009 TOP 25 COMPANIES BY COUNTRYWIDE PREMIUM
By Line of Business
04-Homeowners Multiple Peril

RANK BY PREM	GROUP/ COMPANY CODE	GROUP/COMPANY NAME	DIRECT PREMIUMS WRITTEN	DIRECT PREMIUMS EARNED	DIRECT LOSS TO EP RATIO[1]	DIRECT LOSS & DCC TO EP RATIO[2]	MARKET SHARE %	CUMULATIVE MARKET SHARE %
1	176	STATE FARM GRP	14,737,236,449	14,297,373,262	66.53	68.50	21.62	21.62
2	8	ALLSTATE INS GRP	6,837,285,776	6,862,769,439	60.40	62.40	10.03	31.65
3	212	ZURICH INS GRP	5,066,034,148	5,036,069,846	50.38	51.92	7.43	39.08
4	111	LIBERTY MUT GRP	3,461,188,432	3,316,831,558	56.33	57.76	5.08	44.16
5	3548	TRAVELERS GRP	3,082,154,511	2,983,717,008	47.20	48.56	4.52	48.68
6	200	UNITED SERV AUTOMOBILE ASSN GRP	2,969,697,662	2,806,871,503	55.20	56.29	4.36	53.04
7	140	NATIONWIDE CORP GRP	2,909,147,923	2,879,824,709	64.35	65.72	4.27	57.30
8	38	CHUBB & SON INC GRP	1,772,267,678	1,811,450,950	41.39	39.72	2.60	59.90
9	473	AMERICAN FAMILY INS GRP	1,460,618,362	1,434,633,263	68.24	69.04	2.14	62.05
10	91	HARTFORD FIRE & CAS GRP	1,122,779,740	1,107,932,897	65.91	69.38	1.65	63.69
11	280	AUTO OWNERS GRP	878,807,476	852,604,259	91.93	93.37	1.29	64.98
12	241	METROPOLITAN GRP	860,986,068	851,833,843	52.86	53.27	1.26	66.25
13	213	ERIE INS GRP	792,687,542	765,832,058	67.23	70.77	1.16	67.41
14	10064	CITIZENS PROP INS CORP	790,756,469	882,461,699	41.85	42.37	1.16	68.57
15	12	AMERICAN INTL GRP	715,528,744	502,028,806	53.27	61.34	1.05	69.62
16	761	ALLIANZ INS GRP	647,504,956	657,173,787	37.39	38.37	0.95	70.57
17	50	COUNTRY INS & FIN SERV GRP	577,090,552	559,186,343	87.51	88.57	0.85	71.42
18	4663	UNIVERSAL INS HOLDING GRP	513,340,865	492,282,608	33.80	34.60	0.75	72.17
19	1278	CALIFORNIA STATE AUTO GRP	511,857,303	495,352,273	52.69	54.69	0.75	72.92
20	88	THE HANOVER INS GRP	493,687,770	473,609,354	63.55	64.60	0.72	73.64
21	1318	AUTO CLUB ENTERPRISES INS GRP	463,117,940	444,409,266	45.52	48.59	0.68	74.32
22	28	AMICA MUT GRP	431,265,405	422,793,164	49.25	51.18	0.63	74.96
23	361	MUNICH RE GRP	393,833,029	370,328,757	46.41	46.46	0.58	75.53
24	55	AUTOMOBILE CLUB MI GRP	367,095,292	351,770,261	65.67	66.41	0.54	76.07
25	1344	ARX HOLDING CORP GRP	321,405,260	287,149,029	31.01	33.04	0.47	76.54
		INDUSTRY TOTAL	68,166,767,715	66,459,328,229	59.27	60.94	100.00	100.00

[1] (Direct losses incurred / Direct premiums earned)

[2] (Direct losses incurred + Direct defense and cost containment expenses incurred) / (Direct premiums earned)

Table 1: Statistics of Premiums and Losses of 2009 Top 25 Companies in Property and Casualty Insurance Industry. This table shows the statistics of premiums and losses of 2009 top 25 companies in property and casualty insurance industry, collected by NAIC (National Association of Insurance Commissioners). The ratios of losses to premiums are 59.27% and 60.94%, based on different definitions of losses. The former is defined by direct losses incurred divided by direct premiums earned while the latter is defined by the sum of direct losses incurred and direct defense and cost containment expenses incurred divided by direct premium earned.
Source: http://www.naic.org/research_home.htm

9. Conclusions

This paper compares parimutuels to insurance contracts in a market where risk-averse people seek to hedge. We construct two models where an individual places a parimutuel stake to hedge against potential catastrophes. In the first model, an optimal stake choice is obtained when the total stakes on the target area placed by other participants, and the total stakes outside the target area, are exogenous. The optimal stake can be obtained by equating the marginal cost of a net payoff with the ratio of the expected marginal utilities in the payoff state and the no payoff state. When the odds and the conditional probability of a hurricane hitting the target areas are available, we derive the dynamic optimal hedge rule by the first order condition. In the next model, the equilibrium of the parimutuel stake is derived based on a representative agent's optimal choice. If there is no transaction fee and tax, parimutuel insurance intrinsically leads to underinsurance due to the basis risk, a result that is inconsistent with traditional insurance where the decision maker has background risk. Furthermore, the sensitivities of the equilibrium of the parimutuel stake with respect to the potential loss, the conditional probability of a hurricane hitting the target area if it occurs, and the conditional probability of an individual's asset being destroyed if a hurricane hits the target area, are all positive; however, the sensitivity of the equilibrium with respect to the initial wealth is ambiguous.

WRS invented an option, called HuRLOs, which embodies the concept of parimutuel insurance. Issuers of HuRLOs act as a bookmaker and do not bear the underlying risk, but participants who intend to hedge will have inbuilt basis risk compared to traditional insurance with no transaction cost. However, in the presence of transaction costs, the choice between hedging with a HuRLO or with insurance, essentially boils down to a trade off between transaction costs and basis risk. We derive the equivalent transaction costs such that the expected utility with optimal insurance equates the expected utility yielded by the optimal HuRLO stake. The actual transaction cost for traditional homeowners multiple peril insurance is approximately 67%, higher than the utility equivalent level (33.2%-43.2%) implied by the HuRLOs with minor transaction cost. This empirical result suggests that homeowners who would like to hedge against hurricane risk would be better off with HuRLOs than with traditional insurance because in practice, the transaction cost of HuRLOs is very low relative to traditional insurance.

In this paper, the parimutuel market is analyzed focusing on hedgers. Speculators simply arbitrage on the discrepancy between the forecast-based probability and the market-based probability, and thus have no role to play in equilibrium. However, speculators could deviate from this simple arbitrage, because of irrational behaviors affected by psychological factors, or because they have different belief about the objective probability. Empirical anomalies, such as the favorite-longshot bias, may result from these behaviors of speculators. Taking the actions of speculators into consideration may change the optimal stake choice for hedgers in parimutuels.

Appendix 1

$$\frac{\partial E[U(\widetilde{W})]}{\partial x}$$

$$= \frac{1-p_s}{p_s} \cdot \frac{U'(W-x\ )}{(1-p_i)\cdot U'(W+O_s(x)\cdot x)+p_i\cdot U'(W+O_s(x)\cdot x-L)}$$

$$= \frac{\partial\left(O_s(x)\cdot x\right)}{\partial x}$$

$$= \frac{\partial\left(NI_s(x)\right)}{\partial x}$$

$$= \frac{\partial\left(O_s(x)\right)}{\partial x}\cdot x + O_s(x)$$

$$= \frac{M_0}{m_0+x}\cdot\frac{m_0}{m_0+x}$$

$$\frac{\partial\left[O_s(x)\cdot x\right]}{\partial x} = \frac{m(x)-M(x)}{\left[m(x)\right]^2}\cdot x + \frac{M(x)}{m(x)} - 1 = \frac{M(x)-m(x)}{m(x)}\cdot\frac{m(x)-x}{m(x)}$$

$$= \frac{M_0\cdot m_0}{(m_0+x)^2} > 0$$

$$\frac{\partial^2\left[O_s(x)\cdot x\right]}{\partial x^2} = \frac{-2\cdot\left[m(x)-M(x)\right]}{\left[m(x)\right]^3}\cdot x + 2\cdot\frac{m(x)-M(x)}{\left[m(x)\right]^2}$$

$$= -2\cdot\frac{M(x)-m(x)}{\left[m(x)\right]^2}\cdot\frac{m(x)-x}{m(x)} = -2\cdot\frac{M(x)-m(x)}{\left[m(x)\right]^3}\cdot\left(m(x)-x\right)$$

$$= \frac{-2\cdot M_0\cdot m_0}{(m_0+x)^3} < 0$$

Appendix 2

Rearranging from the F.O.C., we have the following formula:

$$f(W, x^*, p_i, S, L)$$

$$= U'(W - x^*) - (1 - p_i) \cdot U'(W + (S-1) \cdot x^*) - p_i \cdot U'(W - L + (S-1) \cdot x^*) = 0$$

The comparative static of the equilibrium of pari-mutuel stake with respect to four parameters in our model can be derived by Implicit Function Theorem:

$$\frac{\partial x^*}{\partial S} = -\frac{\frac{\partial f}{\partial S}}{\frac{\partial f}{\partial x^*}}, \frac{\partial x^*}{\partial p_i} = -\frac{\frac{\partial f}{\partial p_i}}{\frac{\partial f}{\partial x^*}}, \frac{\partial x^*}{\partial L} = -\frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial x^*}}, \frac{\partial x^*}{\partial W} = -\frac{\frac{\partial f}{\partial W}}{\frac{\partial f}{\partial x^*}}$$

Implicitly differentiating f with respect to the x^*, pi, S, L, and W yields the following equations with their signs:

$$\frac{\partial f}{\partial x^*} = -U''(W - x^*) - (1 - p_i) \cdot (S-1) \cdot U''(W + (S-1) \cdot x^*)$$

$$- p_i \cdot (S-1) \cdot U''(W - L + (S-1) \cdot x^*) > 0$$

$$\frac{\partial f}{\partial p_i} = U'(W + (S-1) \cdot x^*) - U'(W - L + (S-1) \cdot x^*) < 0$$

$$\Rightarrow \frac{\partial x^*}{\partial p_i} = -\frac{\frac{\partial f}{\partial p_i}}{\frac{\partial f}{\partial x^*}} > 0 \Leftrightarrow \frac{\partial x^*}{\partial n} < 0 \left(\because p_i = \frac{1}{n} \right)$$

$$\frac{\partial f}{\partial S} = -(1 - p_i) \cdot x^* \cdot U''(W + (S-1) \cdot x^*) - p_i \cdot x^* \cdot U''(W - L + (S-1) \cdot x^*)$$

$$= -x^* \cdot U''(W + (S-1) \cdot x^*) + p_i \cdot x^* \cdot \underbrace{\left[U''(W + (S-1) \cdot x^*) - U''(W - L + (S-1) \cdot x^*) \right]}_{>0} > 0$$

$$\Rightarrow \frac{\partial x^*}{\partial S} = -\frac{\frac{\partial f}{\partial S}}{\frac{\partial f}{\partial x^*}} < 0 \Leftrightarrow \frac{\partial x^*}{\partial p_s} > 0 \left(\because p_s = \frac{1}{S} \right)$$

$$\frac{\partial f}{\partial L} = p_i \cdot U''(W - L + (S-1) \cdot x^*) < 0$$

$$\Rightarrow \frac{\partial x^*}{\partial L} = -\frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial x^*}} > 0$$

$$\frac{\partial f}{\partial W} = U''(W - x^*) - (1 - p_i) \cdot U''(W + (S-1) \cdot x^*) - p_i \cdot U''(W - L + (S-1) \cdot x^*)$$

$$= \underbrace{\left[U''(W - x^*) - U''(W + (S-1) \cdot x^*) \right]}_{<0}$$

$$+ p_i \cdot \underbrace{\left[U''(W + (S-1) \cdot x^*) - U''(W - L + (S-1) \cdot x^*) \right]}_{>0}$$

$$\Rightarrow \frac{\partial x^*}{\partial W} = -\frac{\frac{\partial f}{\partial W}}{\frac{\partial f}{\partial x^*}} > 0 \Leftrightarrow \frac{\partial f}{\partial W} < 0$$

Since the sign of $\frac{\partial x^*}{\partial W}$ cannot be determined, we further analyze under the following condition the sign is positive.

$$\frac{\partial x^*}{\partial W} > 0 \Leftrightarrow$$

$$U''(W - x^*) - (1 - p_i) \cdot U''(W + (S-1) \cdot x^*) - p_i \cdot U''(W - L + (S-1) \cdot x^*) < 0$$

$$\therefore \frac{\partial x^*}{\partial W} > 0 \Leftrightarrow U''(W - x^*) < (1 - p_i) \cdot U''(W + (S-1) \cdot x^*) + p_i \cdot U''(W - L + (S-1) \cdot x^*)$$

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